

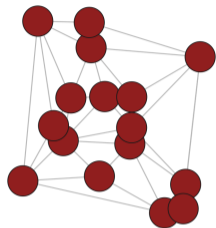
L2G-Net: Local to Global Spectral GNNs via Cauchy Factorizations

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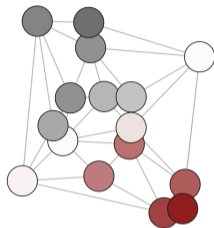
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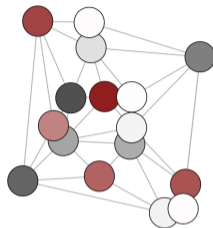
Spectral GNNs and the GFT



$\lambda = 0.00$



$\lambda = 0.33$



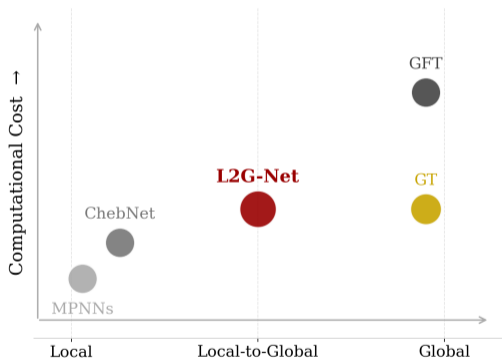
$\lambda = 1.57$

Graph Fourier transform (GFT) [Ortega et al., 2018]: global representations.

Cubic complexity $O(n^3) \implies$ impractical for large graphs.

Fully global \implies no local inductive bias for short-range tasks.

Existing approaches

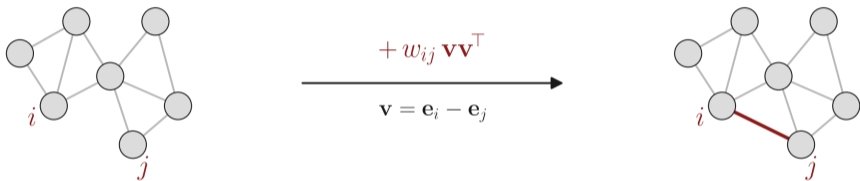


- **MPNNs** [Kipf and Welling, 2017] and **polynomial filters** [Defferrard et al., 2016]: local, but cause oversquashing.
- **Graph transformers** [Dwivedi and Bresson, 2021]: global, but lose topological inductive bias.
- Our goal: **exact GFT** with **local-to-global** processing.

Background: rank-one updates and Cauchy matrices

Adding an edge (i, j) to graph Laplacian \mathbf{L} yields a **rank-one update**:

$$\tilde{\mathbf{L}} = \mathbf{L} + w_{ij} \mathbf{v}\mathbf{v}^\top, \quad \mathbf{v} = \mathbf{e}_i - \mathbf{e}_j. \quad (1)$$

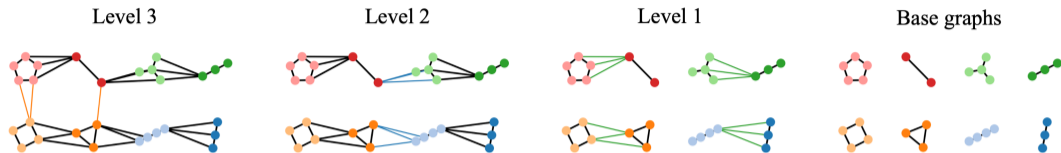


Cauchy update of the eigenbasis [Fernández et al., 2025]

Rank-one update \implies rotate by an **orthogonal Cauchy-like matrix**:

$$\tilde{\mathbf{U}}^\top = \mathbf{C}(\tilde{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) \mathbf{U}^\top. \quad (2)$$

GFT via divide and conquer



- 1 **Partition** the graph into subgraphs (e.g., balanced cuts).
- 2 **Compute** GFT of base graphs.
- 3 Progressively **merge** base GFTs using edge information.

Key: merging is fast via structured Cauchy matrices [Fernández et al., 2025].

Cauchy factorization of the GFT

Theorem 3.1: GFT factorization

For any graph $\mathcal{G} \in \mathcal{F}(L, \{\mathcal{G}_i\}_{i=1}^m, k)$ with GFT basis \mathbf{U}^\top :

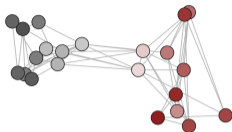
$$\mathbf{U}^\top = \mathbf{D}(\boldsymbol{\lambda}, \tilde{\boldsymbol{\lambda}}_{K-1}) \cdots \mathbf{D}(\tilde{\boldsymbol{\lambda}}_1, \tilde{\boldsymbol{\lambda}}_0) \mathbf{U}_0^\top. \quad (3)$$

Start from **independent subgraph GFTs**, merge one cut edge at a time.

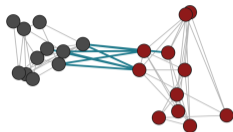
Each Cauchy factor is **localized** to the two subgraphs it connects.

Computable in **quadratic time**: $O(kn^2 + \sum_i f_i(n))$.

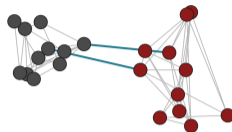
Finding a suitable hierarchy



Fiedler vector



Partition: 8 bridge edges



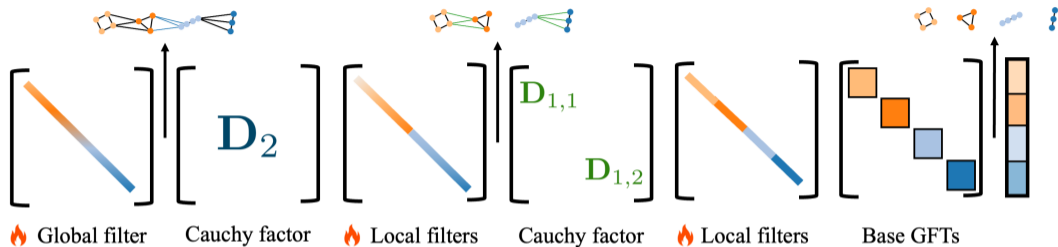
Sparsified: 2 bridge edges

Balanced cuts via spectral bisection (Fiedler vector).

Accept partition only if $n^2k + \max_i f(\mathcal{G}_i) < f(\mathcal{G})$.

Cut sparsification: reduce k via Spielman-Srivastava sparsifier.

L2G-Net architecture

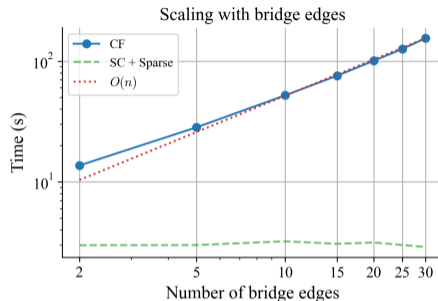
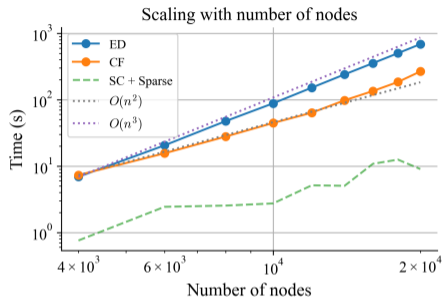


Local filters capture patterns within subgraphs.

Global filter models long-range subgraph interactions.

Architecture **adapts to graph structure** before training begins.

Empirical complexity validation

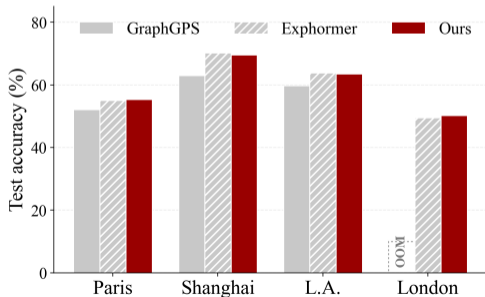
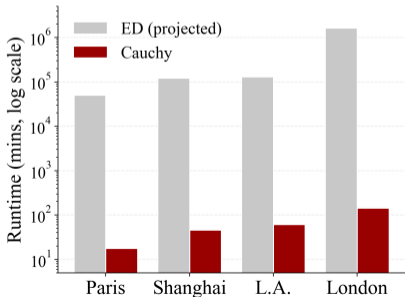
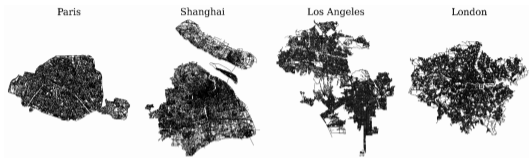


Runtime scales as $O(n^2)$ confirming Theorem 3.2 predictions.

Linear scaling with cut size k : **sparsification yields large savings.**

GNNs in CityNetworks [Liang et al., 2025]

Global: Estimate eccentricity of each node.



Conclusions and future work

L2G-Net: **exact GFT** with **local-to-global** inductive bias.

Cauchy factorization reduces complexity from $O(n^3)$ to $O(kn^2)$.

- Competitive accuracy with far fewer parameters.

Future work:

- Extension to **directed graphs**.
- **Dynamic graphs** via incremental updates.

Questions? samuelf9@usc.edu



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Our contribution

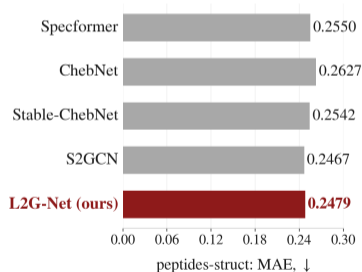
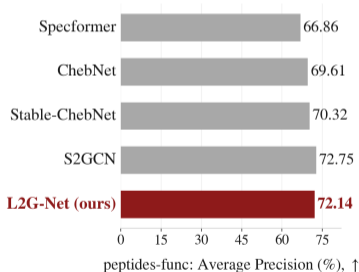
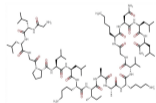
We introduce an **exact factorization** of the GFT via Cauchy matrices.

- 1) **Quadratic complexity** $O(kn^2)$ scaled by the subgraph cut size.
- 2) **Graph partitioning** algorithm to minimize factorization cost.
- 3) **L2G-Net**: a new class of spectral GNNs with local-to-global bias.

GNNs in drug discovery [Dwivedi et al., 2022]

Global: chemical properties depend on **interactions across the whole molecule**.

Local: **functional groups** govern short-range chemistry.



L2G-Net **captures both regimes** with a single architecture.